Mechanisms to Improve Decentralized Category Management with Vendor-Specific Product Portfolios

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Category management is a popular retail practice in which pricing, assortment, and stocking decisions are coordinated across products within a particular retail category such as laundry detergent or toothpaste. Recently some retailers have been delegating the assortment decision to their vendors, but it is not known how well this approach performs. We analyze an industry-motivated model of category management in which the retailer allows each vendor to make stocking and assortment decisions for a given amount of shelf space. We determine the optimal pricing decisions for the retailer and the best stocking and assortment policies for each vendor in a two-stage decentralized system. We find that when the vendors’ stocking incentives run contrary to the retailer’s preferences, the retailer can be considerably worse off (i.e., experience profit losses as high as 40%) by delegating these responsibilities to the vendors. We compare the delegated system to a retailer-controlled channel and a centralized supply chain and demonstrate how a minimum-profit constraint can induce the vendors to achieve the total profit of a retailer-controlled channel but it may not induce centralized performance. To improve the performance of the decentralized, vendor-controlled channel, we show that a revenue-sharing arrangement with a discounted wholesale price is guaranteed to achieve full supply chain coordination in a vendor-controlled channel when the vendors produce multiple substitutable products and shelf space is limited or ample. Revenue sharing may not coordinate a vendor-controlled channel with medium levels of shelf space, but we characterize the difference in the decisions compared with the centralized channel’s decisions, which is small for most parameter realizations. Through our analysis of revenue sharing, we establish that a revenue-sharing contract can coordinate stocking levels in a general, uncapacitated supply chain consisting of multiple vendors setting stocking levels for multiple substitutable products.

1 Introduction

Faced with ever-shrinking margins, increased competition from brick-and-mortar stores and online e-tailers, and increasing demands from customers, most retailers are engaged in a constant search for ways to reduce their costs while at the same time increasing their level of customer service. This journey has led many retailers, especially grocers, to the potential savior known as category
management. Although the term has become an umbrella moniker that can include many different practices and goals, all definitions of category management emphasize the joint management and planning for an entire category of products (e.g., soft drinks, laundry detergent, or socks) instead of individual stock-keeping units (SKUs) with the goal of improving the availability, pricing, product assortment, and timing of promotions with respect to customer demand (Dupre and Gruen, 2004). A report by Cannondale Associates estimates that retailers have experienced a 14-percent growth in sales and manufacturers a corresponding 8-percent increase in sales as a result of category management practices (Raskin, 2003).

Category management efforts may differ with respect to which firm is responsible for performing the analysis and making the recommendations. Some retailers such as Giant Eagle (Tortola, 2004), The County Grocer (Felix, 2006), and Spartan Stores (Garry, 2005) employ their own personnel to manage their product categories. Other retailers, though, have allocated category management responsibilities to their vendors. The main rationale for allowing the vendors to make the category recommendations is that individual retailers cannot be experts in all product segments. According to Pat Garvey, manager of retail floor space for apparel supplier VF, “No matter how expert [retailers] are in their area, they can’t be an expert on every single brand a lot of the time” (Ryan, 2004). The retailer also saves the cost and effort involved in managing a category, which can be considerable, by allocating this responsibility to her vendors. As a former category manager put it, “As a retailer, you get the best minds in the business working for you free of charge. Why not take advantage of that?” (Raskin, 2003).

One form of vendor-controlled category management is category captainship, in which one vendor in a category is designated as the “captain” who is responsible for making the category decisions for his products as well as any competitors’ products. There are many examples in practice of this type of arrangement working well, but it is obvious that there is an inherent potential in this kind of structure for the captain vendor to recommend his products to the detriment of his major competing vendor. Category captainship arrangements have also come under scrutiny as a potential violation of antitrust legislation. The Federal Trade Commission (2001) has identified three types of antitrust violation that can be facilitated by category management: (1) obtaining confidential information about competitors’ plans, (2) competitive exclusion of other vendors’ products, (3) fostering collusion between retailers or vendors. Category captainship arrangements typically vio-
late antitrust law if they restrict competition or reduce consumer utility (Desrochers et al., 2003). Several high-profile court decisions in the smokeless tobacco, cigarette, and tortilla markets have deemed particular category captain programs in violation of antitrust statutes; there are cases currently pending involving the sale of cranberries and soft drinks as well (Desrochers et al., 2003; Raskin, 2003). The governments of the United States and Canada have each issued reports that identify category captainship as a business practice of anticompetitive concern, and Israel has required governmental approval for category management initiatives undertaken by its three largest retailers (Desrochers et al., 2003).

One way to mitigate antitrust concerns is to allow each vendor to make decisions about its own products, a practice that some retailers have used with great success. Therefore, we analyze an alternative form of category management called vendor-specific category management (VSCM), in which each vendor has the responsibility of managing the stocking and assortment decisions for its own shelf space provided by the retailer. VSCM-type arrangements have been used successfully by apparel companies such as VF and Gold Toe (Ryan, 2004; DesMarteau, 2004). The retailer is still able to avoid the cost and effort required by category management, and she benefits from having the vendors, who are the experts in their particular product category, make the shelf-space management recommendations. The potential agency problem of having a captain vendor make recommendations about other vendors’ products is eliminated; thus, the anticompetitive concerns about the practice should be reduced, if not removed completely. It is not clear, however, what effects these types of arrangements have on profits and consumer utility in competitive markets.

The major contribution of our study is the analysis of a decentralized category management mechanism in which each vendor controls his own space and stocking levels at the retailer while the retailer sets the retail prices. We characterize the subgame-perfect Nash Equilibrium decisions made by each party in a two-stage supply chain game with two vendors and a single retailer. Our focus is on analyzing these decisions to capture insights about when this decentralized approach is beneficial and what strategies are effective in improving the channel’s performance. When the parties’ preferences as to which products to stock under circumstances of extremely limited shelf space are aligned, VSCM naturally performs close to the optimal retailer-controlled channel; however, when the incentives are misaligned, the retailer can be considerably worse off (profit loss as high as 40%) than if he retained the assortment decision. We compare the VSCM mechanism
with a traditional retailer-controlled channel as well as a centralized supply chain. We show that a minimum-profit constraint can achieve the performance of a retailer-controlled channel, but it can be costly to set up and may not achieve full channel coordination. In response, we demonstrate that a revenue-sharing contract can be effective in coordinating the supply chain in many cases. We add to the current research on revenue-sharing contracts by showing that revenue sharing coor-
dinates the channel when shelf space is ample and when vendors can offer multiple products under
deterministic demand.

2 Literature Review

Category management has its origins in the study of methods for shelf-space allocation. Corstjens
and Doyle (1981) develop a model where the main driver of the assortment decision is the relation-
ship between the amount of shelf space assigned to each product and the store’s profitability by
incorporating direct shelf-space elasticities and cross-product space elasticities. More recent studies
(e.g., van Ryzin and Mahajan (1999) and Cachon et al. (2002)) have utilized a multinomial logit
(MNL) model of consumer choice in determining a retailer’s optimal assortment strategy. Chong et
al. (2001) use an MNL consumer choice model and develop three brand-width measures that incor-
porate the homogeneity and heterogeneity of products within a manufacturer’s product portfolio
and across manufacturers into the assortment decision. Hall et al. (2003) study a fixed-horizon
dynamic pricing and ordering model of the retailer’s category management decision. In each of the
studies above, the retailer retains the assortment decision, and none of them consider the supply
chain implications for the vendors of using these models for shelf-space management.

Another relevant group of studies model the supply chain effect of centralized brand manage-
ment at the retailer. Zenor (1994) develops a model in which vendors can choose to create a coalition
with their products when they offer wholesale prices to retailers with no space considerations, and
Martín-Herrán et al. (2006) consider the effect of the vendors’ wholesale prices on the retailer’s re-
sulting shelf-space allocation. Basuroy et al. (2001) develop a competitive market with two vendors
each producing a single product and two retailers. They examine the equilibria characteristics that
arise when the retailers practice traditional, individual brand management or centralized category
management. Moorthy (2005) determines the effect of retailer competition and individual-brand
management versus category management strategies at two competing retailers on how different
cost changes are allocated through the channel. In all of these studies, the assortment decision still rests with the retailer; whereas, these responsibilities are delegated to the vendors in the VSCM model we study.

The papers that are most related to our study analyze scenarios of category captainship, in which the retail assortment decision rests with one of the vendors. In all of these models, the retailer adopts the captain’s recommendation in all circumstances; in practice, however, some retailers do not necessarily implement all of the captain’s suggestions (Raskin, 2003). Kaipia and Tanskanen (2003) present a case study in which the shelf-space assortment is kept close to optimal when the vendor manages the store shelves. Kurtulus and Toktay (2005) consider category captainship with two vendors each producing a single product and one retailer. One of the vendors is chosen as the captain and is responsible for managing a given amount of retailer shelf space subject to a minimum profit constraint dictated by the retailer. They find that in some situations the non-captain vendor’s product is excluded from the assortment under category management when it would not be if the retailer retained the assortment decision. Kurtulus and Toktay (2006) examine the effect that category captainship has on the variety experienced by consumers and compare the effect of three types of retailer targets (profit, sales, and variety). Wang et al. (2003) focus mainly on the appropriate choice of a category captain vendor who will control the pricing and allocation decisions for the entire category. Our VSCM model allows each vendor to manage his own retailer-allocated shelf space (instead of a single captain vendor making decisions for the entire category) while the retailer retains price control, and we develop methods for coordinating the supply chain.

Since the retailer transfers control of its shelf space to its vendors, our model can be thought of as an extension of traditional vendor-managed inventory (VMI) methods. VMI is a well-studied technique for reducing the bullwhip effect in serial supply chain in which the retailer gives control over his stocking levels to the vendors to mitigate the effects of order batching for the vendors, to reduce his inventory control costs, and to allow the vendor to coordinate production and distribution across multiple retailers. See Mishra and Raghunathan (2004), Choi et al. (2004), and Bernstein et al. (2007) for examples of the latest research on VMI methods. Our VSCM model takes traditional VMI one step farther by allowing the vendors to select the product assortment at the retailer in addition to the stocking level.

Many methods exist for achieving the centralized supply chain profit in a decentralized channel
by eliminating the double-marginalization effect of individual decision makers. The revenue-sharing contract is a well-studied mechanism for channel coordination. Perhaps the most famous industry application of revenue sharing is in the video rental industry (see Dana and Spier (2001) for details). Giannoccaro and Pontrandolfo (2002), Cachon and Lariviere (2005), and Bernstein and Federgruen (2005) all develop supply-chain-coordinating models utilizing revenue sharing with a single supplier and a single or multiple retailers. Luo and Çakanyıldırım (2005) establish that a revenue-sharing contract can be Pareto-improving for both parties in a VMI system. In this paper we show that when the vendors make decisions on product assortment and stocking levels rather than setting wholesale prices, revenue sharing can improve the performance of our category management channel in many cases. In all of the above revenue-sharing models, the manufacturer is assumed to produce only one product; we add to the existing literature on revenue sharing by showing that revenue-sharing contracts can also coordinate the supply chain when multiple vendors offer substitutable products in a market segment.

The remainder of the paper is organized as follows. The next section develops the vendor-specific category management model and establishes the retailer-control and centralized benchmark models. Section 4 discusses two benchmark cases and compares the performance of the vendor-specific category management system with these benchmarks. Several methods of aligning the parties' incentives are analyzed in Section 5, and Section 6 provides concluding remarks and suggestions for future research in this area.

3 Vendor-Specific Category Management (VSCM) Model

Let us consider a two-stage supply chain for a single product category consisting of two vendors and one retailer. There are many examples of markets in which firms produce many substitutable products within a single product category including toothpaste, laundry detergent, beer, and athletic shoes. Substitution is a key factor in stocking decisions within a category. To focus on providing insights for this issue, we consider a market in which Vendor 1 manufacturers two products for the category and Vendor 2 produces a third; all three products are substitutes for each other within the particular category. We expect some of our main insights to extend naturally to larger category markets with many vendors and products. We analyze a single-period system in order to isolate the decision drivers without confounding the results with the effects of multi-period inventory control.
The system we analyze can be thought of as a single period of a multi-period process.

The serial decision structure under VSCM is as follows. The retailer determines the shelf spaces, $S_1$ and $S_2$ (measured in units of product), she will make available to Vendor 1 and Vendor 2, respectively. Many retailers such as Dollar General, Foot Locker, and CVS operate small stores in which the shelf space allocated to each product category is limited; consequently, the effective use of this space is of paramount concern to the retailer and her vendors. In our model we assume that the vendor space-allocation decision has been made \textit{a priori}, but it is also possible to consider this decision as an additional decision stage within our framework. This assumption is consistent with some current retail practices in which a retailer “sells” the space to the vendor either explicitly through slotting allowances or indirectly by providing some level of promotional effort to induce a certain level of sales (Klein and Wright, 2004).

Each vendor simultaneously determines the quantity of its good(s) to stock at the retailer subject to the allocated space and declares this to the retailer. The retailer then simultaneously determines the prices at which to sell her vendor-prescribed inventory to maximize her profit. Some retailers such as Dollar General retain pricing control in their interactions with their vendors. Since the retailer interacts directly with the final customers, she can easily and quickly adjust the retail price in response to the vendors’ supply quantities. The retailer maintaining the pricing decision can also be viewed as a method for validating the vendors’ stocking decisions. Even if the vendors make recommendations about the retail prices in practice, the retailer will only adopt these prices if they confirm their appropriateness. By allowing the retailer to set the prices in our VSCM model, we explicitly include the fact that she must approve of whatever prices are ultimately charged; consequently, the retailer controls the quantity that she wants to sell. Note that this is an important difference in our model compared to models of category captainship in which the retailer adopts the captain’s pricing recommendations completely. In order to prevent the vendors from dumping unwanted inventory on the retailer, we make the following assumption which is consistent with the models of consignment-type VMI discussed in Bernstein et al. (2007).

\textbf{Assumption 1} \textit{The retailer only pays the vendor for units that she ultimately sells (similar to a buyback contract at full price).}

We will denote the two different products manufactured by Vendor 1 as Products 1 and 2. The product made by Vendor 2 will be known as Product 3. The wholesale prices, production costs, and
Table 1: Summary of model notation

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_i$</td>
<td>Wholesale price (per unit) paid by the retailer to the vendor producing product $i$</td>
</tr>
<tr>
<td>$c_i$</td>
<td>Production cost (per unit) paid by the vendor producing product $i$</td>
</tr>
<tr>
<td>$q_i$</td>
<td>Number of units of product $i$ supplied by its manufacturing vendor</td>
</tr>
<tr>
<td>$p_i$</td>
<td>Retail price (per unit) charged by the retailer for product $i$</td>
</tr>
<tr>
<td>$S_j$</td>
<td>Shelf space (measured in units) allocated by the retailer to vendor $j$, $j = 1, 2$</td>
</tr>
<tr>
<td>$D_i(p)$</td>
<td>Consumer demand for product $i$ as a function of retail prices, $p$</td>
</tr>
</tbody>
</table>

Retailer shelf space allocations are exogenous to our decision environment. In our model one could think of the wholesale price being determined by a fixed markup over cost or by a long-term contract as in many inventory stocking problems; retail prices are often much more malleable in short-term, day-to-day operations in response to shifting consumer preferences. Fixing the wholesale prices also allows us to study the vendors’ stocking preferences without the confounding influence of the wholesale pricing decision.

The notation for the model is summarized in Table 1. The vendors select the $q_i$ amounts that they will stock at the retailer, and the retailer chooses the vector of prices, $p = (p_1, p_2, p_3)$, that she will charge the end consumers.

Following the tradition in the marketing, economics, and operations literature (see, e.g., Levitan and Shubik (1971), McGuire and Staelin (1983), and Biller et al. (2006).), we utilize linear demand functions with price substitution effects in our analysis. This type of demand function results from an underlying consumer utility maximization problem and captures the substitution characteristics of traditional downward-sloping demand; namely, the quantity demanded of each product is decreasing in its own price and increasing in the prices of the other goods. See Kurtuluş and Toktay (2005) for a derivation of this demand function from the representative consumer utility model in Shubik and Levitan (1980). This deterministic demand structure allows us to isolate the effect of the supply chain incentives created by the vendor-specific category management strategy without having the results confounded with the effects of demand variability. The end-user demand functions for each of the three products are as follows.

$$D_1(p) = a_1 - b_1p_1 + \beta_{12}p_2 + \beta_{13}p_3$$
$$D_2(p) = a_2 - b_2p_2 + \beta_{21}p_1 + \beta_{23}p_3$$
$$D_3(p) = a_3 - b_3p_3 + \beta_{31}p_1 + \beta_{32}p_2$$
All of the parameters comprising the demand functions are assumed to be non-negative, which ensures that the goods are substitutes. The substitution effects result in complex equilibrium decisions; so for readability, we use compact notation for these decisions in discussion of the results and provide the full decisions in the Appendix. Complete information exists between all of the parties, so each player knows the others’ costs, space allocations, and the system of demand functions.

The following assumption will allow us to establish concavity of the retailer’s profit function. It requires that each good’s own-price effect is twice-as-large as any of the cross-price effects; price increases for competing products will cause some consumers to substitute other products, but each of the other product brands will only experience a fraction of the total substitution purchases because they are in competition with each other.

Assumption 2 Each of the $b_i$ terms is greater than the sum of product $i$’s cross-price effects with one of the other two products. More succinctly, we have $b_i \geq \beta_{i,j} + \beta_{j,i}$ for all $i = 1, 2, 3$ and $j \neq i$.

The final assumption we make is that Vendor 1’s products have profit margins that are sufficiently “close.” We want to consider situations where Vendor 1 would choose to stock both products for medium levels of shelf space rather than stock the maximum amount of his high-margin product that he could sell and leave empty shelf space. The latter situation is not expected to occur in practice because in a retail environment with shelf-space limitations, the retailer would reallocate the unused space to another vendor. If this assumption did not hold, then the reduction in sales of Product 1 required to offer Product 2 would outweigh the gain from selling Product 2.

Assumption 3 Without loss of generality, Product 1 is the more profitable product for Vendor 1. The profit margins are such that $w_2 - c_2 \geq \rho(w_1 - c_1)$, where

$$\rho = \frac{b_3(\beta_{12} + \beta_{21}) + \beta_{13}\beta_{23} + \beta_{13}\beta_{22}}{2(b_2b_3 - \beta_{23}\beta_{32})}.$$

3.1 Retailer’s Pricing Decision

Since we have a two-stage dynamic game of perfect information, we will solve for the equilibrium strategies using backward induction. We begin, therefore, by examining the retailer’s optimal pricing best response given that each of the vendors has provided $q_i$ of their respective products.\footnote{Since the demand functions are deterministic, we could equivalently solve for the retailer’s optimal selling quantities subject to the supply quantities received from each vendor.}

The retailer is faced with the following profit maximization problem.

$$\max_{p_1, p_2, p_3} \sum_{i=1}^{3} (p_i - w_i) D_i(p)$$
\[ s.t. \quad 0 \leq D_i(p) \leq q_i \quad i = 1, 2, 3 \]
\[ p_1, p_2, p_3 \geq 0 \]

**Lemma 1** The retailer’s objective function in her profit maximization problem is concave in the retail prices.\(^2\)

Note that the retailer may choose not to sell her entire supply if doing so would significantly degrade her resulting retail price. Since the objective function is concave, and we have constraints that are linear in the decision variables (retail prices), we can solve the retailer’s problem using the Karush-Kuhn-Tucker (KKT) conditions (c.f. Bazaraa et al. (1993: 151–55)). Because of the structure of the demand functions induced by Assumption 2, the retailer would never choose to sell a negative quantity or offer a product at a negative price even if the non-negativity constraints were not present in her problem. This leaves us with four possible retailer scenarios related to the supply provided by the vendors, which we discuss below.

1. **None of the products are constrained.** The retailer has enough units to supply her simultaneously-unconstrained profit-maximizing quantities. We solve for the prices that make the gradient of the profit function equal to zero to obtain the set of optimal prices, \(\hat{p}_i\) for \(i = 1, 2, 3\), which is given in equations (3)–(5) in the Appendix. We will denote the retailer’s corresponding unconstrained profit-maximizing sales quantities as \(\hat{D}_i = D_i(\hat{p}_1, \hat{p}_2, \hat{p}_3)\) for \(i = 1, 2, 3\).

2. **One of the products is priced such that demand equals supply.** The retailer sells her entire stock of one product and has enough supply to sell her resulting desired amount of the other two products. For example, suppose that the retailer sells out of Product 2, which means that \(D_2(p) = q_2\). Scenario 2 has two analogous scenarios in which Product 1 and Product 3 are the only ones sold out, respectively. The prices can be found by switching all of the “2” subscripts with either “1” or “3,” depending on which product is sold out. This enables us to solve for one of the prices as a function of the other prices and \(q_2\). We can then equate the gradient of the Lagrangian with zero by solving for the other two prices and the KKT multiplier for the supply constraint for Product 2 to obtain \(p_i(q_2)\) for \(i = 1, 2, 3\) given in (6)–(8) in the Appendix. The resulting customer demands, \(Q_i(q_2)\) for \(i = 1, 2, 3\) are given in (9)–(11).

\(^2\)The proofs of all lemmas and theorems are provided in the Online Appendix.
3. Two of the products are priced such that demand equals supply. The retailer sells her entire stock of two products and has enough supply to sell her resulting desired amount of the third product. For example, suppose that the retailer sells out of Products 2 and 3; thus, $D_2(p) = q_2$ and $D_3(p) = q_3$. Scenario 3 also has two analogous scenarios in which each of the other two products replaces Product 1 as the unconstrained good. As in Scenario 2 the prices and sales quantities for these cases are found by replacing all “1” subscripts with “2” and “3,” respectively. We can solve these two equations for two of the retail prices and then solve for the third price and the two KKT supply constraint multipliers that make the gradient of the Lagrangian equal to zero to obtain $p_i(q_2, q_3)$ for $i = 1, 2, 3$ given in (12)–(14) in the Appendix. The resulting customer demands are given in (15)–(17).

4. All of the products are priced such that demand equals supply. The retailer sells her entire stock of all three products. The optimal retail prices are obtained by solving the three $D_i(p) = q_i$ equations, which yields (18)–(20) in the Appendix. The resulting customer demands for each product are the $q_i$ values we used in determining the corresponding prices.

The notation we use to represent the unconstrained customer demands shows explicitly that they are a function of the constrained supply of various products. For example, $Q_1(q_2)$ is meant to denote the unconstrained demand for Product 1 when exactly $q_2$ units of Product 2 will be sold.

In order to determine which of the four scenarios is valid for a particular vector of supply quantities, $q = (q_1, q_2, q_3)$, one could look at the values of the KKT multipliers and see if they are non-negative. The structure of the optimal multipliers in our problem, however, is prohibitively complex for meaningful analysis. Fortunately, we can equivalently determine which scenario is applicable by computing the optimal customer sales quantities in each of the four scenarios and determining which set of quantities satisfies all of the constraints in the retailer’s profit maximization problem. We will use this observation to establish the constraints for the vendors’ decision problems discussed in the next section.

### 3.2 Vendors’ Shelf-Space Stocking Decisions

Now that we have characterized the retailer’s best response to any set of supply quantities, we can incorporate this into the vendors’ shelf-space optimization problems. Note that since each vendor has control over his own shelf space, we have an optimization problem for each vendor that the
two vendors solve simultaneously. The fact that Vendor 2 only produces one good simplifies this simultaneous analysis so that we can provide insights about Vendor 1’s stocking incentives.

Each of the vendors seeks to maximize his profit subject to the shelf space that the retailer has allocated him. His profit function changes, however, according to the pricing actions taken by the retailer in response to the quantities of products he supplies. Consequently, there are several different vendor cases, each of which induces a separate retailer scenario from the set described in the previous section. We provide the vendors’ (simultaneous) profit maximization problems for each of these scenarios in Table 2.³

The optimization problems in Table 2 differ in the expressions for the quantity of each product for which the vendors will receive revenue from the retailer as well as the shelf space constraints that determine the ranges of supply quantities in which this case will be applicable. For example, in Case IV the retailer is constrained in the supply of Product 2, and she is able to sell her corresponding unconstrained amounts of Products 1 and 3. (Note that this corresponds to Retailer Scenario 2 discussed in Section 3.1.) As a result, Vendor 1 receives revenue for all $q_2$ units of Product 2 that he supplies since the retailer will sell the entire amount, but he will only earn revenue on a maximum of $Q_1(q_2)$ units of Product 1. Vendor 1 prefers to stock Product 1 over Product 2, so he will always supply a quantity of Product 1 equal to $Q_1(q_2)$ in this case and we have adjusted his profit function to reflect this policy. Likewise, in this case Vendor 2 earns revenue on a maximum of $Q_3(q_2)$ units of Product 3, and his profit function reflects this. Each of the profit-maximization problems for the other vendor cases can be interpreted analogously where the demand quantities determine the constraints. Examination of the optimization problems in the six vendor cases yields the following observation.

**Observation 1** *In any case where the vendor’s profit function contains the retailer’s sales quantity for his revenue and his supply amount ($q_i$) for his costs, the vendor will only supply the amount that the retailer will sell.*

Observation 1 is an easily-seen result of Assumption 1. Each vendor’s revenue is based on what the retailer ultimately sells, so it does him no good to supply more units that the retailer would

³The optimization problems for Vendor 1 in Cases III and IV are written to reflect that Product 1 is the more profitable product for Vendor 1 to sell. Each of these cases has an analogous scenario (denoted III.B and IV.B) in which Product 2 has the higher margin. These problems can be written by switching all of the “1” subscripts in the original problems to “2” and vice versa.
Table 2: Vendors’ profit maximization problems in each vendor scenario

**Case I: All products constrained**

\[
\begin{align*}
\text{Vendor 1:} &\quad \max_{q_1,q_2} (w_1 - c_1)q_1 + (w_2 - c_2)q_2 \\
\text{s.t.} &\quad q_1 + q_2 \leq S_1 \\
&\quad q_1 \leq Q_1(q_2, q_3) \\
&\quad q_2 \leq Q_2(q_1, q_3) \\
\text{Vendor 2:} &\quad \max_{q_3} (w_3 - c_3)q_3 \\
\text{s.t.} &\quad q_3 \leq S_2 \\
&\quad q_3 \leq Q_3(q_1, q_2)
\end{align*}
\]

**Case II: Vendor 2 unconstrained**

\[
\begin{align*}
\text{Vendor 1:} &\quad \max_{q_1,q_2} (w_1 - c_1)q_1 + (w_2 - c_2)q_2 \\
\text{s.t.} &\quad q_1 + q_2 \leq S_1 \\
&\quad q_1 \leq Q_1(q_2) \\
&\quad q_2 \leq Q_2(q_1) \\
\text{Vendor 2:} &\quad \max_{q_3} w_3Q_3(q_1, q_2) - c_3q_3 \\
\text{s.t.} &\quad q_3 \leq S_2 \\
&\quad q_3 \geq Q_3(q_1, q_2)
\end{align*}
\]

**Case III: Product 1 unconstrained and Products 2 and 3 constrained**

\[
\begin{align*}
\text{Vendor 1:} &\quad \max_{q_1,q_2} w_1Q_1(q_2, q_3) - c_1q_1 + (w_2 - c_2)q_2 \\
\text{s.t.} &\quad q_1 + q_2 \leq S_1 \\
&\quad q_1 \geq Q_1(q_2, q_3) \\
&\quad q_2 \leq Q_2(q_1, q_3) \\
\text{Vendor 2:} &\quad \max_{q_3} (w_3 - c_3)q_3 \\
\text{s.t.} &\quad q_3 \leq S_2 \\
&\quad q_3 \leq Q_3(q_1, q_2)
\end{align*}
\]

**Case IV: Products 1 and 3 unconstrained and Product 2 constrained**

\[
\begin{align*}
\text{Vendor 1:} &\quad \max_{q_1,q_2} w_1Q_1(q_2) - c_1q_1 + (w_2 - c_2)q_2 \\
\text{s.t.} &\quad q_1 + q_2 \leq S_1 \\
&\quad q_1 \geq Q_1(q_2) \\
&\quad q_2 \leq \hat{D}_2 \\
\text{Vendor 2:} &\quad \max_{q_3} w_3Q_3(q_1, q_2) - c_3q_3 \\
\text{s.t.} &\quad q_3 \leq S_2 \\
&\quad q_3 \geq Q_3(q_2)
\end{align*}
\]

**Case V: Products 1 and 2 unconstrained and Product 3 constrained**

\[
\begin{align*}
\text{Vendor 1:} &\quad \max_{q_1,q_2} w_1Q_1(q_3) - c_1q_1 + w_2Q_2(q_3) - c_2q_2 \\
\text{s.t.} &\quad q_1 + q_2 \leq S_1 \\
&\quad q_1 \geq Q_1(q_3) \\
&\quad q_2 \geq Q_2(q_3) \\
\text{Vendor 2:} &\quad \max_{q_3} (w_3 - c_3)q_3 \\
\text{s.t.} &\quad q_3 \leq S_2 \\
&\quad q_3 \leq \hat{D}_3
\end{align*}
\]

**Case VI: All products unconstrained**

\[
\begin{align*}
\text{Vendor 1:} &\quad \max_{q_1,q_2} w_1\hat{D}_1 - c_1q_1 + w_2\hat{D}_2 - c_2q_2 \\
\text{s.t.} &\quad q_1 + q_2 \leq S_1 \\
&\quad q_1 \geq \hat{D}_1 \\
&\quad q_2 \geq \hat{D}_2 \\
\text{Vendor 2:} &\quad \max_{q_3} w_3\hat{D}_3 - c_3q_3 \\
\text{s.t.} &\quad q_3 \leq S_2 \\
&\quad q_3 \geq \hat{D}_3
\end{align*}
\]
choose to sell. This is one of the built-in incentive alignment mechanisms in our vendor-specific category management model compared with the models of category captainship. The fact that the retailer retains her pricing decision and only pays for the units she sells in our model provides a degree of safety against the negative impacts of vendor assortment decisions for the retailer and corresponds with typical VMI arrangements. These negative impacts could occur when the vendor and retailer’s profit margins induce them to stock different products; we refer to this environment as having misaligned stocking incentives.

**Observation 2** *In all of the cases, the vendors’ profit functions are linear in the supply quantity decision variables; thus, the profits are strictly increasing or decreasing with the decision variables. Consequently, the equilibrium solution is determined by identifying the set of tight constraints.*

Observation 2 suggests a methodology that we will use in determining the vendors’ equilibrium strategies. Since we know that at least one of the constraints will be tight, we can substitute the values into vendor 1’s profit function and determine from the sign of the derivative if he wants \( q_1 \) or \( q_2 \) to be large or small. Then we can solve explicitly for the shelf-space conditions that ensure that the vendors’ decision values are valid in the current scenario. The solution approach is presented in the Online Appendix, and the results, assuming Product 1 generates the higher margin for the vendor, are summarized in Table 3.

**Observation 3** *Since Vendor 1’s profit function is linear in his two supply quantities in all cases, as Vendor 1’s shelf space increases he prefers to stock solely the highest margin product for as long as possible.*

Observation 3 implies that stocking Product 1 is always preferable to stocking Product 2 for Vendor 1 as long as the retailer chooses to sell the quantity he supplies. Any shelf-space allocations that have a given proportion of Product 1 will always generate a higher profit for Vendor 1 than any allocation that includes fewer units of Product 1. This has major implications if the vendor is responsible for selecting the allocation.

The vendor cases in Table 2 exhibit a natural ordering that allows us to develop the spectrum of applicable cases depicted in Figure 1 as a function of the vendors’ shelf-space allocations. Consider the cases where Vendor 2’s shelf space is small and \( q_3^* = S_2 \). Since Vendor 1 is making the
Table 3: Equilibrium decision sets and validity conditions for each vendor scenario

<table>
<thead>
<tr>
<th>Case</th>
<th>Decisions</th>
<th>Conditions</th>
</tr>
</thead>
</table>
| I.   | \( q_1^* = S_1 \)  
     | \( q_2^* = S_2 \)  
     | \( q_3^* = S_2 \)  
     | \( S_1 \leq Q_1(q_2 = 0, q_3 = S_2) \)  
     | \( S_2 \leq Q_3(q_1 = S_1, q_2 = 0) \)  |
| II.  | \( q_1^* = S_1 \)  
     | \( q_2^* = 0 \)  
     | \( q_3^* = Q_3(q_1 = S_1, q_2 = 0) \)  
     | \( S_1 \leq Q_1(q_2 = 0) \)  
     | \( S_2 \geq Q_3(q_1 = S_1, q_2 = 0) \)  |
| III. | \( q_1^* = S_1 - q_2^{III} \)  
     | \( q_2^* = q_2^{III} \)  
     | \( q_3^* = S_2 \)  
     | \( S_1 \geq Q_1(q_2 = 0, q_3 = S_2) \)  
     | \( S_1 \leq Q_1(q_3 = S_2) + Q_2(q_3 = S_2) \)  
     | \( S_2 \leq Q_3(q_1 = S_1 - q_2^{III}, q_2 = q_2^{III}) \)  |
| IV.  | \( q_1^* = S_1 - q_2^{IV} \)  
     | \( q_2^* = q_2^{IV} \)  
     | \( q_3^* = Q_3(q_2 = q_2^{IV}) \)  
     | \( S_1 \geq Q_1(q_2 = 0) \)  
     | \( S_1 \leq \hat{D}_1 + \hat{D}_2 \)  
     | \( S_2 \geq Q_3(q_1 = S_1 - q_2^{IV}, q_2 = q_2^{IV}) \)  |
| V.   | \( q_1 = Q_1(q_3 = S_2) \)  
     | \( q_2 = Q_2(q_3 = S_2) \)  
     | \( q_3 = S_2 \)  
     | \( S_1 \geq Q_1(q_3 = S_2) + Q_2(q_3 = S_2) \)  
     | \( S_2 \leq \hat{D}_3 \)  |
| VI.  | \( q_1 = \hat{D}_1 \)  
     | \( q_2 = \hat{D}_2 \)  
     | \( q_3 = \hat{D}_3 \)  
     | \( S_1 \geq \hat{D}_1 + \hat{D}_2 \)  
     | \( S_2 \geq \hat{D}_3 \)  |

assortment decision, he wants to sell as many units of Product 1 (his highest-margin product) as he can. Consequently, he will fill his shelf space with Product 1 if space is limited \((S_1 \leq Q_1(q_2 = 0, q_3 = S_2))\). This corresponds with Vendor Case I. Once his space, \( S_1 \), exceeds the boundary where Case I is valid, Assumption 3 ensures that he will start stocking some of Product 2. This amount, though, will only be enough so that Product 1’s unconstrained sales amount exactly fills up the space \((i.e., q_2 + Q_1(q_2, q_3 = S_2) = S_1)\); thus, the decision environment moves to Case III. When his space is very large, then Vendor 1 supplies the resulting unconstrained sales amount when \( q_3 = S_2 \), which represents a move to Case V. When Vendor 2 has a large amount of space, Vendor 1 faces a similar spatial decision spectrum only with Vendor 2 supplying unconstrained quantities of Product 3. We use the equilibrium decisions to analyze the performance of the system as a whole.

4 Vendor-Specific Category Management’s Performance Relative to Benchmark Systems

In this section we investigate the performance characteristics of the VSCM system compared to a traditional, retailer-controlled (RCM) channel as well as the efficient, centralized supply chain. In
the RCM channel, the retailer determines both the stocking levels and the product assortment on her shelves. To ensure that we compare analogous systems, we assume that the retailer has already assigned $S_1$ units of space to Vendor 1’s products and $S_2$ units to Vendor 2’s single product.

### 4.1 Analysis of the Retailer-Controlled Channel

In a RCM channel, the retailer seeks to determine her profit-maximizing quantities of each product to purchase from the vendors subject to the shelf-space constraints. It is clear that the retailer will purchase only such quantities that she is able to sell since we are considering a market with deterministic demand. Consequently, the retail prices will be those from Retailer Scenario 4 (discussed in Section 3.1), where the retailer sells out of her supply of all of the products. Recall that these prices are denoted $p_i(q_1, q_2, q_3)$. We can now write the retailer’s profit-maximization problem in a RCM channel as

$$
\max_{q_1, q_2, q_3} \sum_{i=1}^{3} (p_i(q_1, q_2, q_3) - w_i)q_i
$$

s.t.

$$
q_1 + q_2 \leq S_1
$$

$$
q_3 \leq S_2
$$

$$
q_1, q_2, q_3 \geq 0.
$$

The retailer’s problem has the same decision structure as in the VSCM channel. That is, there are six decision regions corresponding the vendor cases depicted in Figure 1. Since the retailer is concerned with maximizing her profit (and not the vendors’), it is clear that the optimal decision
values will be different from those in the VSCM system, which are still feasible in the retailer-controlled channel. This means that the retailer must be worse off by allowing the vendors to manage their own product assortments. The issue, though, is capturing the magnitude of the retailer’s loss. The retailer may still be willing to adopt a VSCM system if her profit loss is small enough to be offset by the gain from not having to exert the effort (and cost) of managing the category assortment and stocking levels. As we will show with several numerical examples, the degree of profit loss for the retailer is dependent on the natural alignment of each party’s incentives concerning the preferential product to stock when Vendor 1’s space is extremely limited as well as the initial allocation of shelf space to each vendor. Misaligned stocking incentives can make VSCM very costly for the retailer.

The optimal stocking strategy for the RCM channel also has a range of space values in which only one of Vendor 1’s products is stocked. Assuming that consumers value the option to purchase different brands, we can compare these single-product ranges under the VSCM and RCM schemes. In many consumer products’ industries, consumers possess strong brand allegiance to particular products. Kim et al. (2002) suggest that each brand be thought of as an imperfect substitute for the other brands in order to represent the fact that some households would suffer an extreme loss in utility from the exclusion of a particular brand. In addition, retailers may be the only source of particular household goods (e.g., toothpaste or laundry detergent) in economically-depressed areas. (See Lavin (2005) for a case study of the establishment of a Pathmark supermarket store in the Harlem section of New York City.) If these retailers do not stock particular brands, then the consumers who live in these areas will have a limited amount of brand choice, thus decreasing their utility.

The following theorem shows that the consumers can be worse off with respect to their product variety availability under VSCM than under RCM when the parties’ single-product stocking preferences are aligned. The main implication of the theorem is that the allocation of category management decisions can have an impact not only on profit, but also on consumer welfare.

**Theorem 1** When Vendor 2’s shelf space is small \( q_3^* = S_2 \) and when the retailer and Vendor 1 prefer to stock the same initial product, the space level under retailer-controlled category man-

\[\text{\textsuperscript{4}}\text{A similar result was evident in all the numerical examples we have seen when the parties’ preferences for single-product stocking are different, but we do not expect the result to hold in general.}\]
Table 4: Parameter declarations for numerical example with misaligned incentives

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_1$</td>
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<td>$b_1$</td>
<td>5</td>
<td>$c_1$</td>
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<td>$a_3$</td>
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<td>4</td>
<td>$c_3$</td>
<td>1.25</td>
</tr>
<tr>
<td>$\beta_{12}$</td>
<td>0.9</td>
<td>$\beta_{23}$</td>
<td>0.75</td>
<td>$w_1$</td>
<td>4</td>
</tr>
<tr>
<td>$\beta_{13}$</td>
<td>0.8</td>
<td>$\beta_{31}$</td>
<td>0.7</td>
<td>$w_2$</td>
<td>2</td>
</tr>
<tr>
<td>$\beta_{21}$</td>
<td>0.85</td>
<td>$\beta_{32}$</td>
<td>0.95</td>
<td>$w_3$</td>
<td>3</td>
</tr>
<tr>
<td>$w_1$</td>
<td>4</td>
<td>$w_2$</td>
<td>2</td>
<td>$w_3$</td>
<td>3</td>
</tr>
</tbody>
</table>

The result in Theorem 1 that the retailer prefers to increase the product assortment at lower levels of space runs contrary to a result of the category captainship model presented in Kurtuluş and Toktay (2005). They find that the category captain makes decisions that can reduce the product variety available to the customers and that the retailer prefers to have more product homogeneity than the captain when she is deciding to add another product to the assortment. In our case both vendors make an assortment decision, and the retailer prefers more variety than the vendors since she chooses to include an additional product to the category assortment at a lower level of shelf space. The fact that consumers can be worse off under VSCM makes the design of incentive-coordinating mechanisms even more important to consider in instances where Vendor 1 prefers to stock only one product.

4.2 Performance of Vendor-Specific Category Management with Misaligned Incentives

In this section we provide a numerical example in which the single-product stocking preferences of Vendor 1 and the retailer are mismatched to compare the VSCM system to a RCM channel and a centralized supply chain. In particular, Vendor 1 would like to stock as much as Product 1 as possible in small shelf-space areas, while the retailer prefers stocking Product 2. This is obviously the case in which the retailer stands to lose the most from allowing the vendors to control her shelf space, and we find that indeed the retailer’s profit loss can be quite severe.

The model parameters utilized in this example are presented in Table 4. (The graph of the six vendor cases in Figure 1 was generated from this example.) It is clear that Vendor 1 prefers stocking
Product 1, but the retailer’s single-product stocking preference is more difficult to establish. To determine which product the retailer prefers to stock first, one looks at the margin she receives at minimal stocking levels (where zero units of both products are stocked). For any value of \( q_3 \), the optimal retail prices are \( p_1(q_1 = 0, q_2 = 0, q_3) = 9.2039 - 0.0608q_3 \) and \( p_2(q_1 = 0, q_2 = 0, q_3) = 9.8450 - 0.0876q_3 \). The maximum amount of product 3 that the retailer can sell when neither of the other two products are stocked is \( Q_3(q_1 = 0, q_2 = 0) = 10.5686 \), and at this maximum value for \( q_3 \), the retail price of Product 2 is larger than Product 1. Coupled with the fact that Product 1 has a higher wholesale price, this confirms that the retailer prefers to stock Product 2 first. Since the production costs exhibit the same ordering as the wholesale prices, the centralized firm also prefers to stock Product 2 solely.

The retailer’s profit under VSCM as a function of the shelf space allocated to the two vendors is provided in Figure 2(a). The shelf-space boundaries at which the slope of the profit function changes drastically correspond to the space conditions at which the VSCM channel switches cases. For large values of \( S_1 \) and \( S_2 \), the profit curve plateaus, representing the fact that the retailer prefers to leave extra space at these high levels. Note also that the retailer’s profit is non-decreasing in each shelf-space value since the retailer should never be worse off when he has more space (because she could always choose not to sell the entire supply she receives as a result of retaining retail pricing power).
The retailer’s net benefit from adopting the VSCM channel structure comprises gains due to delegation of the cost and effort required to manage the category that are offset by the profit loss from accepting the vendors’ stocking and assortment decisions. Since the retailer would only initiate a VSCM structure if she received a positive net benefit, we examine the performance of the VSCM channel relative to the (presumed) status quo of RCM in Figure 2(b) to determine the potential magnitude of the profit loss. We see that the retailer can earn up to 40% less profit under the VSCM system. The profit loss is most severe when Vendor 2 has little shelf space and Vendor 1 has a “medium” level of space because Vendor 1 chooses to fill this relatively-large amount of space with Product 1 when the retailer would rather have more of Product 2. When Vendor 1’s space is small, the effect of the incentive misalignment is mitigated somewhat by the fact that Vendor 1 only has a little bit of space to stock in the first place. When Vendor 1 has no space or a very large amount of space, the VSCM approaches or reaches full efficiency either because Vendor 1’s decision set reduces or the parties’ interests naturally align when space is not a constraining factor.

Figures 3(a) and 3(b) depict the vendors’ profits as a function of the two shelf-space values. As we would expect, each vendor’s profit is non-decreasing in his own shelf space and is non-increasing in the other vendor’s space. Each vendor’s profit function has its steepest slope when the other vendor has a small amount of space. This stems from the fact that the vendor can take advantage of the customers’ willingness to purchase substitutes if the price of the other product is high.
In Figure 2(b) we saw that the retailer can be significantly worse off under a VSCM channel. Some of this lost profit, though, is captured by the vendors through their ability to select product stocking levels and assortments. Consequently, the comparison of the total supply chain profit under VSCM and the optimal centralized profit in Figure 4 shows that the maximum efficiency loss in this example is approximately 20%, which is still quite significant. The two efficiency graphs illustrate that incentive realignment is especially crucial in cases where the first vendor and the retailer prefer different single-product stocking assortments.

4.3 Performance of Vendor-Specific Category Management with Naturally-Aligned Incentives

In this section we consider a numerical example in which the retailer and Vendor 1 both prefer the same single product in small shelf-space areas. To make the example as close as possible to the previous example, we use all of the same parameter values in Table 4 except that \( a_1 = 60 \) (instead of 30). This adjusts the retailer’s initial marginal profit for Product 1 to \( p_1(q_1 = 0, q_2 = 0, q_3) - w_1 = 11.8989 - 0.0608q_3 \) and to \( p_2(q_1 = 0, q_2 = 0, q_3) - w_2 = 10.1730 - 0.0876q_3 \) for Product 2. The margin for Product 1 is larger than that of Product 2 for any value of \( q_3 \), so the retailer (and the centralized channel) now prefers to stock Product 1 in areas of small shelf space. Since the rest of the parameters are the same as in the previous example, the vendor’s product preference is still for Product 1.
Our main interest is in the effect of this natural incentive alignment on the retailer’s efficiency with respect to a RCM channel and the total supply chain efficiency relative to the centralized channel. The retailer’s efficiency maintains the same graphical form, but her maximum profit loss is approximately 10%, compared with 40% in the previous case. Thus, formal incentive alignment is less important from the retailer’s perspective if the vendor chooses the same product to stock solely; the retailer be willing to accept such profit losses if she can avoid the administrative and analytical costs of category management. The total supply chain profit is close to optimal as well, with the maximum efficiency loss being less than 6%. Interestingly, the decentralized VSCM system behaves like a de facto centralized system when both vendors have small space allocations since Vendor 1 fills his entire shelf space with the same product that the centralized system would choose. In the example with misaligned stocking incentives, the VSCM system does not achieve total efficiency because Vendor 1 is stocking the “wrong” product from a supply chain perspective.

5 Methods of Incentive Coordination

As we saw in Section 4.2, the retailer can experience a significant loss in profit from allowing the vendors to control their shelf space compared with what she could earn from retaining that decision in a RCM channel. Consequently, the retailer would likely want to introduce either performance standards or an incentive-realignment mechanism to adjust the vendors’ decentralized performance to a level in accordance with her interests. Following our discussions with industry about a mechanism currently in place in one VSCM system, we first investigate the imposition of a minimum-performance threshold to control the vendors’ actions.

Theorem 2 The retailer can induce Vendor 1 to stock her optimal RCM sales quantities in a VSCM channel by imposing a minimum-profit requirement.

At first glance, it seems as if this constraint does an adequate job in aligning the parties’ decisions. Technically, it works fine, but it is somewhat unsatisfying for practice because by dictating the profit that she must earn, the retailer is actually determining the stocking quantities that she will receive. She may be nominally allocating the assortment decision to the vendors, but the vendors do not really have control over the decision since they have to stock the retailer-controlled optimal quantities in order to satisfy the performance standard. One way to determine the profit
requirements is to use the profit the retailer was achieving when she managed the stocking decisions, but this may be ineffective when new products are introduced to the category. The retailer is still incurring the costs required to calculate the appropriate profit requirements for the constraint and is not gaining any benefit. Further, this mechanism may not achieve centralized channel performance because the retailer’s pricing decision is based on product margins that do not correspond with those of the centralized channel. In light of these shortcomings, we investigate the adoption of a revenue-sharing contract structure in this environment to allow the vendors to retain responsibility for the stocking and assortment decisions both nominally and practically.

**Theorem 3** A revenue-sharing contract in which the retailer shares a \((1-\alpha)\) fraction of its revenue from each product to the product’s manufacturer and the vendors charge wholesale prices, \(w_i = \alpha c_i\) for \(i = 1, 2, 3\), results in a VSCM channel that attains the same profit as the optimal centralized supply chain when Vendor 1’s shelf space is extremely limited or suitably ample. When vendor shelf space is middling, revenue sharing may be Pareto improving (especially if Vendor 1’s single-product stocking incentives differ from the centralized channel’s preferences), but it is not guaranteed to be effective in all cases.

Theorem 3 establishes an extension of the classic revenue-sharing contract to guarantee coordination of a channel with multiple substitutable products and multiple vendors, albeit with deterministic demand. Unlike the minimum-performance requirement options above, a revenue-sharing mechanism does not subject the vendors to additional constraints. It adjusts the profit functions that they optimize in determining their stocking quantities. For a specific value of \(\alpha\), one of the vendors or the retailer can be worse off under VSCM than he would be under a traditional RCM channel since the \(\alpha\) parameter determines how to split the optimal supply chain profit. In order to develop a strictly Pareto-improving solution for all parties, a particular \(\alpha\) could be chosen or the party that benefits the most could share some of their gain with the adversely-affected party via a lump-sum side payment.

The revenue-sharing mechanism is not necessarily able to achieve channel coordination for medium-sized levels of shelf space. This deficiency stems from the fact that in this case the retailer will sell all of the units of each product that the Vendor 1 supplies. She cannot use her pricing decision as a credible threat to induce the vendor to provide the centralized quantities as she can when shelf space is ample because the capacity is limited enough that she is still willing to sell
the non-coordinated amounts. When capacity is very limited, the centralized channel stocks one product in Vendor 1’s space, and Vendor 1 follows suit because he also prefers to stock only one product in the small area. The vendor’s profit function under medium-sized shelf space, however, is sufficiently “close” to the centralized channel so that his stocking quantities will not vary too much from the optimal centralized quantities (as demonstrated in the following theorem).

**Theorem 4** In the cases where the revenue-sharing contract fails to coordinate the VSCM channel and Vendor 2 provides $q_3$ units of Product 3, Vendor 1’s optimal supply quantities differ from the optimal centralized quantities by

$$
\pm q_3 \left( \frac{b_2 \beta_{31} + \beta_{21} \beta_{32}}{b_1 b_3 - \beta_{13} \beta_{31}} - 2(b_3 \beta_{12} + \beta_{13} \beta_{32}) \right).
$$

This difference in the optimal stocking levels in Theorem 4 is not very large for most parameter realizations satisfying Assumption 2. It is especially small if the products have similar substitution characteristics with each other and similar own-price elasticities. The difference is a result of the profit function of the retailer (and the centralized channel) including the price of Product 3, which is also affected by the stocking levels of Products 1 and 2. Vendor 1’s profit function does not have this extra term; thus, it does not appear in his optimal quantities. In these cases Vendor 2 may not supply the centralized quantity of Product 3, but he will provide the centralized channel’s best response to the stocking levels provided by Vendor 1 since his incentives are aligned with those of the centralized channel.

For an illustration of Theorem 4, we apply a revenue-sharing mechanism to the example in Section 4.2 with misaligned single-product stocking incentives. Consider a scenario in which Vendor 1 has four units of shelf space and Vendor 2 has five units of space. In a VSCM channel without revenue sharing, Vendor 1 fills his entire shelf space with Product 1, yielding a total supply chain profit of 54.34. The optimal RCM decisions are $q^{RCM} = (0.1768, 3.8234, 5)$, which yields a supply chain profit of 57.00 (44.07 for the retailer, 4.18 for Vendor 1, and 8.75 for Vendor 2). If a revenue-sharing contract with $\alpha = 0.84$ is employed, the optimal VSCM stocking quantities are $q^{VSCM} = (1.0282, 2.9718, 5)$, which results in a supply chain profit of 57.53. The retailer now earns a profit of 48.33, Vendor 1 realizes 4.47, and Vendor 2 obtains 4.73. The retailer can provide a lump-sum payment to Vendor 2 in order to increase his profit above 8.75 while still retaining profit higher than 44.07; this results in a strictly-Pareto improving solution over RCM for all parties. Other values
of $\alpha$ will also achieve the same supply chain profit, but one of the parties may have to provide a lump-sum payment to achieve a strictly-Pareto improving solution. The maximal centralized channel profit is 57.56 (obtained from optimal stocking quantities $q^{Central} = (1.3036, 2.6964, 5)$), which shows that revenue sharing attains performance very close to the centralized channel in this example. The revenue-sharing VSCM channel generates total supply chain profit within 0.05% of the centralized channel while earning 5.87% more supply chain profit than a VSCM channel with no incentive alignment mechanism.

The rationale used in the proof of Theorem 3 for environments with large shelf space says that the first vendor has an incentive to overstock each of his products relative to the quantities that the retailer prefers to sell when the other vendor supplies the centralized quantities. Consequently, the vendor’s best response is to stock the centralized quantities as well because he will not be paid for any units that the retailer fails to sell. It seems that an analogous argument would apply in a market with multiple vendors each producing multiple products. This leads us to conjecture that a revenue-sharing agreement similar to that discussed in Theorem 3 would also coordinate a VSCM channel in markets with more vendors and products and no shelf-space considerations.

6 Conclusions and Future Research

In this paper we have analyzed the performance of a type of decentralized category management channel called vendor-specific category management (VSCM) that is currently being used in the apparel industry. The VSCM structure is different from models of category captainship that allocate the stocking and assortment decisions at the retailer to a single, “captain” vendor; under VSCM each vendor is responsible for maintaining the stocking and assortment of its own products in a designated area of shelf space at the retailer. Since each vendor controls his own space, the VSCM structure eliminates (or dampens) many of the risks of competitive exclusion and antitrust violations inherent under category captainship.

We apply backwards induction to a two-stage supply chain game to find the optimal retailer pricing decisions that define scenarios within which the vendor optimizes. When the wholesale prices and vendor shelf-space allocations are set in advance, we find that the vendor who manufactures two goods chooses to stock one product exclusively in small areas of shelf space and then gradually increases the stock of the second product as he gains more space at the retailer. In some cases the
decentralized channel will stock fewer types of products under VSCM as compared to a retailer-controlled (RCM) system; thus, the consumers’ utilities suffer because they have fewer product options to choose from. In our experiences the retailer’s profit under VSCM can be as much as 40% lower than her profit in RCM, which is consistent with losses from double marginalization. We show that when shelf space is sufficiently small or large, a revenue-sharing arrangement is effective in coordinating the channel with respect to a centralized supply chain. Under medium levels of shelf space, revenue sharing may not fully coordinate the system, but the gap in the equilibrium decisions may be small, especially when the own-price effects and substitution parameters for the products are similar. Our findings also imply that revenue sharing is guaranteed to coordinate a general decentralized system in which the vendors choose stocking levels for multiple substitutable products with deterministic demand.

The fact that many retailers are utilizing vendor-controlled category management practices and the comparatively few academic studies of the effects of these arrangements on supply chain performance suggest that this area provides many opportunities for future research. Since the vendors are controlling the stocking decisions, they should be able to realize efficiencies from coordinating production and distribution across products and customers. These efficiencies could lead the vendors to adjust their stocking decisions in particular periods. In our model we assumed that the retailer’s initial shelf-space allocations for each vendor had been determined exogenously. It would be interesting to study this allocation decision in light of the VSCM model and how it could potentially change over time with all of the vendors’ assortment decisions. It is also important to verify that our results hold under nonlinear and/or stochastic demand processes as well as to investigate the use of category management arrangements in larger-scale systems with many vendors producing many substitutable products as well as the effects of information sharing or information asymmetry on the parties’ decisions.

An overwhelming consensus in the trade literature about category management is that a necessary requirement for successful implementation of category management is the existence of collaboration and trust between the vendor(s) and the retailer. Too often, though, the vendors and the retailer have disparate, if not conflicting, goals for their relationship (Orgel, 2004). According to Tom Fox, a category management consultant, “A big part of category management is collaboration, and I don’t think many retailers do that very well” (Russo, 2004). Further empirical research
on facilitating category management relationships in the vein of Lindblom and Olkkonen’s (2006) study of the effect of vendors’ market power on their ability to make category decisions would help eliminate these practical impediments to the true implementation of aligned category management mechanisms that always have the end-customer’s interests at their core.

Acknowledgments

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7 Appendix: Retailer’s optimal prices and quantities for the four retailer scenarios

Retailer Scenario 1: None of the products are constrained

\[ p_1 = \frac{H_1(4b_2b_3 - (\beta_{21} + \beta_{32})^2) + H_2(2b_2(\beta_{12} + \beta_{21}) + (\beta_{13} + \beta_{31})(\beta_{23} + \beta_{32})) + H_3(2b_1(\beta_{33} + \beta_{31}) + (\beta_{12} + \beta_{21})(\beta_{23} + \beta_{32}))}{2|4b_1b_2b_3 - b_1(\beta_{23} + \beta_{32})^2 - b_2(\beta_{13} + \beta_{31})^2 - b_3(\beta_{12} + \beta_{21})^2 - (\beta_{12} + \beta_{21})(\beta_{13} + \beta_{31})(\beta_{23} + \beta_{32})|} \] (3)

\[ p_2 = \frac{H_1(2b_3(\beta_{12} + \beta_{21}) + (\beta_{13} + \beta_{31})(\beta_{23} + \beta_{32})) + H_2(4b_1b_3 - (\beta_{13} + \beta_{31})^2) + H_3(2b_1(\beta_{33} + \beta_{31}) + (\beta_{12} + \beta_{21})(\beta_{13} + \beta_{31}))}{2|4b_1b_2b_3 - b_1(\beta_{23} + \beta_{32})^2 - b_2(\beta_{13} + \beta_{31})^2 - b_3(\beta_{12} + \beta_{21})^2 - (\beta_{12} + \beta_{21})(\beta_{13} + \beta_{31})(\beta_{23} + \beta_{32})|} \] (4)

\[ p_3 = \frac{H_1(2b_3(\beta_{12} + \beta_{21}) + (\beta_{13} + \beta_{31})(\beta_{23} + \beta_{32})) + H_2(2b_1(\beta_{33} + \beta_{32}) + (\beta_{12} + \beta_{21})(\beta_{13} + \beta_{31})) + H_3(4b_1b_2 - (\beta_{12} + \beta_{21})^2)}{2|4b_1b_2b_3 - b_1(\beta_{23} + \beta_{32})^2 - b_2(\beta_{13} + \beta_{31})^2 - b_3(\beta_{12} + \beta_{21})^2 - (\beta_{12} + \beta_{21})(\beta_{13} + \beta_{31})(\beta_{23} + \beta_{32})|} \] (5)

where \( H_1 = a_1 + b_1w_1 - \beta_{21}w_2 - \beta_{31}w_3, \ H_2 = a_2 + b_2w_2 - \beta_{12}w_1 - \beta_{32}w_3, \) and \( H_3 = a_3 + b_3w_3 - \beta_{13}w_1 - \beta_{23}w_2. \)

Retailer Scenario 2: One of the products is constrained

\[ p_1(q_2) = \frac{K_1(2b_2(b_2b_3 - \beta_{23}32)) + K_2b_2(b_2(\beta_{13}31) + \beta_{21}32 + \beta_{23}32)) + K_2(2b_2(b_2b_3 - \beta_{23}32) + b_2(b_332 + \beta_{31}23) + \beta_{23}(b_232 + \beta_{23}32))}{2(b_2b_3 - \beta_{23}32)(2b_1b_2 - \beta_{12}32) - (b_232 + \beta_{12}23)^2 - 2b_232(b_232 + \beta_{31}23) - (b_232 + \beta_{12}32)(b_232 + \beta_{21}32)} \] (6)
resulting customer demands are

\[ p_2(q_2) = \frac{a_2 - q_2}{b_2} + \frac{K_1(2b_2b_3\beta_{21} + b_2\beta_{23}(\beta_{13} + \beta_{31}) + \beta_{23}^2\beta_{12} - \beta_{21}\beta_{23}\beta_{32}) + K_2(2b_3\beta_{21}^2 + \beta_{21}\beta_{31}\beta_{23} + b_1\beta_{23}^2 + \beta_{13}\beta_{21}\beta_{23}) + K_3(2b_1b_2\beta_{23} + b_3\beta_{21}(\beta_{13} + \beta_{31}) + \beta_{21}\beta_{23}^2 - \beta_{12}\beta_{21}\beta_{23})}{2(b_2b_3 - \beta_{23}\beta_{32})(2b_1b_2 - \beta_{12}\beta_{21}) - (b_2\beta_{13} + \beta_{12}\beta_{23})^2 - 2b_2\beta_{12}(b_2\beta_{31} + \beta_{31}\beta_{23}) - (b_2\beta_{31} + \beta_{21}\beta_{32})(b_2\beta_{31} + 2b_2\beta_{13} + \beta_{21}\beta_{32})} \]  

\[ p_3(q_2) = \frac{K_1(b_2(b_3\beta_{13} + \beta_{21}\beta_{32} + \beta_{23}\beta_{12}) + K_2(2b_1b_2\beta_{12} - \beta_{12}\beta_{21}) + K_3b_2(b_1 - \beta_{12}\beta_{21}) + K_4b_2(b_1b_2 + \beta_{13}\beta_{21} + \beta_{21}(b_2\beta_{31} + \beta_{21}\beta_{32}))}{2(b_2b_3 - \beta_{23}\beta_{32})(2b_1b_2 - \beta_{12}\beta_{21}) - (b_2\beta_{13} + \beta_{12}\beta_{23})^2 - 2b_2\beta_{12}(b_2\beta_{31} + \beta_{31}\beta_{23}) - (b_2\beta_{31} + \beta_{21}\beta_{32})(b_2\beta_{31} + 2b_2\beta_{13} + \beta_{21}\beta_{32})} \]  

where \( K_1 = H_1 + \frac{(a_2 - q_2)(\beta_{12} + \beta_{21})}{b_2} \), \( K_2 = H_2 - 2(a_2 - q_2) \), and \( K_3 = H_3 + \frac{(a_2 - q_2)(\beta_{23} + \beta_{32})}{b_2} \). The resulting customer demands are

\[
Q_1(q_2) = \frac{a_1b_2 + \beta_{12}(a_2 - q_2)}{b_2}
\]

\[
Q_2(q_2) = \frac{a_3b_2 + \beta_{32}(a_2 - q_2)}{b_2}
\]

Retailer Scenario 3: Two of the products are constrained

\[
p_1(q_2, q_3) = \frac{H_1(b_2b_3 - \beta_{23}\beta_{32}) + H_2(b_3\beta_{21} + \beta_{31}\beta_{23}) + H_3(b_2\beta_{31} + \beta_{21}\beta_{32}) + (a_2 - q_2)(b_3\beta_{12} - \beta_{13}\beta_{23} - \beta_{21}\beta_{32} + \beta_{23}\beta_{32} - \beta_{12}\beta_{21})}{2(b_1b_2 - \beta_{12}\beta_{21}) - \beta_{12}(b_3\beta_{21} + \beta_{31}\beta_{23}) - \beta_{13}(b_2\beta_{31} + 2b_2\beta_{13} + \beta_{21}\beta_{32})}
\]
The resulting customer demands are

\[ Q_1(q_2, q_3) = \frac{(a_1 - b_1 w_1)(b_2 b_3 - \beta_{23} \beta_{32}) + (a_2 + b_2 w_1)(b_3 \beta_{12} + \beta_{13} \beta_{32}) + (a_3 + \beta_{31} w_1)(b_2 \beta_{13} + \beta_{12} \beta_{32}) - q_2(b_3(\beta_{12} + \beta_{21}) + \beta_{13} \beta_{32} + \beta_{31} \beta_{23}) - q_3(b_2(\beta_{13} + \beta_{31}) + \beta_{12} \beta_{23} + \beta_{21} \beta_{32})}{2(b_2 b_3 - \beta_{23} \beta_{32})} \]  

\[ Q_2(q_2, q_3) = q_2 \]  

\[ Q_3(q_2, q_3) = q_3. \]

**Retailer Scenario 4: All of the products are constrained**

\[ p_1(q_1, q_2, q_3) = \frac{(a_1 - q_1)(b_2 b_3 - \beta_{23} \beta_{32}) + (a_2 - q_2)(b_3 \beta_{12} + \beta_{13} \beta_{32}) + (a_3 - q_3)(b_2 \beta_{13} + \beta_{12} \beta_{32})}{b_1(b_2 b_3 - \beta_{23} \beta_{32}) - \beta_{12}(b_3 \beta_{21} + \beta_{31} \beta_{23}) - \beta_{13}(b_2 \beta_{31} + \beta_{21} \beta_{32})} \]  

\[ p_2(q_1, q_2, q_3) = \frac{(a_1 - q_1)(b_2 \beta_{21} + \beta_{23} \beta_{32}) + (a_2 - q_2)(b_1 b_3 + \beta_{13} \beta_{31})}{b_1(b_2 b_3 - \beta_{23} \beta_{32}) - \beta_{12}(b_3 \beta_{21} + \beta_{31} \beta_{23}) - \beta_{13}(b_2 \beta_{31} + \beta_{21} \beta_{32})} \]  

\[ p_3(q_1, q_2, q_3) = \frac{(a_1 - q_1)(b_2 b_3 - \beta_{23} \beta_{32}) + (a_2 - q_2)(b_1 b_3 + \beta_{13} \beta_{31})}{b_1(b_2 b_3 - \beta_{23} \beta_{32}) - \beta_{12}(b_3 \beta_{21} + \beta_{31} \beta_{23}) - \beta_{13}(b_2 \beta_{31} + \beta_{21} \beta_{32})} \]

**References**


